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Two further Shapiro effects – "Shapiro drag" and wavelength dependency

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The textbook "Shapiro time delay" associates a gravitational field with an additional time-delay in a signal passing through the region. This should in turn be associated with two further effects: a wavelength dependency for sufficiently long wavelengths, and a drag effect if the gravity-source moves.

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Table of Contents

1. Introduction

The idea that a gravitational well describes the increase in the amount of "effective" space in a region due to gravity, leading to the testable prediction that a gravitational field should be associated with a time-delay in lightsignals passing through the well's region, is credited to **Irwin Shapiro**, who published a paper on the effect in 1964. [2] [3] [4] [5] Shapiro was partly inspired by the analogy between the behaviour of a region of reduced lightspeed due to gravitational time dilation (*as in* Einstein, 1911 [6]) and a region of reduced lightspeed due to a particulate medium and refractive index. [i] [ii] [7]

While Shapiro's observation may have seemed trivial, it had the advantage of being almost immediately testable using contemporary radar and satellite technology, [8] and the "Shapiro timelag" became accepted as Shapiro had referred to it, as "**The Fourth Test of General Relativity**". [3]

Here, we argue for two further extensions of Shapiro's analogy between gravitational physics and the physics of particulate media: a dragging effect similar the to Fresnel-Fizeau drag, [iii] and a wavelength-dependency effect (for wavelengths larger that the distortion-source's characteristic scale).

These similarities beg the question of whether the behaviours of particulate media (often explained using quantum mechanics) can also be modelled classically using generalised geometrical methods developed for general relativity. [iv] [v] [vi]

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i We can also draw parallels with the predictions of C19th aether theories such as that of **George Stokes**, which described an "excess of aether" condensed in matter, and had this aetheric excess extending outward from the Earth by thousands of miles (to give a form of relativity via aether-dragging, *e.g.* **Hertz** 1890 [9]).

ii **Isaac Newton**'s 1704 *Opticks* [10] had previously drawn a direct comparison between deflection due to variations in the speed of light due to the density of matter in a particulate medium, and a gravitational field. Unfortunately, Newton's scalar-based analysis inverted the lightspeed differentials in both situations, [12] so that the speed of light would have to be faster in a more intense gravitational field, and correspondingly faster in a denser particulate medium. Newton's 1704 system … uncorrected … would have generated an **inverse** Shapiro effect. [12] Correcting the inversion gives a positive Newtonian Shapiro effect (**Pössel** 2019 [13]), apparently half of the full Einstein effect, due to the use of curved space but not curved time. Since Einstein's generic 1911 time-dilation argument [6] was explained in the context of Newtonian theory (for simplicity), this argument can presumably then also be invoked to justify a doubling of the predicted "Newtonian" effect. [7]

iii At a late point in the preparation of this paper we came across the Samuel paper. [14] However the treatment of the effect in Samuel's paper is rather different from the approach here.

iv This would require a few modifications to current GR. To agree with QM we would need to adopt an **acoustic metric** [15] with observer-dependent **relative horizons**, rather than absolute horizons. [16] With the emission of photons modelled as Hawking radiation events, we would need to unite the statistics of Hawking radiation with the statistics of conventional photon emission. Such a theory would be pleasing in that while we have already applied the statistical behaviours of particulate media to gravity (analogue gravity, [17] acoustic metrics), this exercise would then apply the resulting QM-compatible classical gravitational equations back to the behaviours of individual particles and their fields. Such a theory would unite QM and (modified) GR by bypassing special relativity. [18]

v This paper does not attempt to unite gravitational and atomic theory. It merely adds two missing pieces to gravitational theory that appear to *correspond* to particulate-media effects, and further strengthens the arguments for gravitomagnetic drag of matter and light by a moving gravitational field-source. Since the **principle of equivalence** of inertia and gravitation makes *every* inertial mass a gravitational field-source, this result means that every inertial mass ought to drag light.

vi The intimately-related case of *timelag due to rotational dragging*, for a signal skimming one side of a star that is stationary but non-static (a star with no overall relative velocity but is *rotating*, and creating a *rotational* drag), is explored by **Ben-Salem and Hackman** (**2022**). [19] Again, light near to a section of stellar surface or horizon that is moving away from us creates a longer timelag than one that is approaching.

2. The "Shapiro drag" effect

2.1 Modelling approach

We can choose to (crudely) model the space surrounding a star as consisting of two components: **(a)** the quantity of space expected if the region was flat and the star absent, and **(b)** the additional quantity of apparent "excess space" due to the star's presence, and its associated spacetime curvature. [i] [ii] It is this second quantity that can be blamed for the Shapiro effect, in which it takes light longer to traverse a region of the region contains a gravitational field due to a star or planet, than it would if the object was not there, and the region was "flat".

2.2 Gravity without gravity

This additional space reveals itself in the form of additional time-lags on signals sent across the region, and these time-lags can be *modelled or simulated* by assuming a region that is *initially* flat but then populated by hypothetical transponders, each set to absorb a signal and rebroadcast it with a set time-delay. We can then recreate an approximation of the light-geometry of the gravitational well, either by saying that the transponders are evenly distributed but that those closest to the star are set to show a longer time-delay, or by having a fixed delay-time-per-transponder, but describing the gravitational field as describing the transponder-density. [iii]

If the transponders are represented as **virtual particles**, we then have a crude form of **quantum gravity**, that can at least describe **scalar gravity**.

2.3 Vectors

The model just described also contains **vector** information, in that if the transponders are considered to be stationary with respect to the star whose field they represent, then if we move relative to the star (or if the star moves relative to us), the transponders have a velocity in our system of reference.

The result of this relative velocity is that the position at which a transponder receives a signal and the later position from which it rebroadcasts it, are different, offset by the distance that the transponder moves between absorption and re-emission. [iv]

i This additional space is present due to the geometry of the star's gravity-well.

ii Current theory actually describes a signal taking longer to cross a "gravitational" region for *two* reasons, an increase in the density of space, and a decrease in the density of time (gravitational time dilation). However, if we are not concerned with the rates of clocks and are solely interested in how long it takes an EM signal to cross the region (without caring "*why*"), then as far as a map of the signal flight-times is concerned, treating both the spatial and temporal distortions as a single combined change in the amount of *apparent* space (as mapped out by lightbeams) still makes sense.

iii In a more extreme version of this type of exercise we could even replace *space itself* with a topological network of connections, and their time-delays (Wheeler's "*pregeometry*", or "*space without space*"). [20]

iv If the transponder is set to have a response time t, and moves at velocity v, then the additional distance travelled due to its motion is $\pm d$, where $d=vt$.

2.4 Lightspeed offset due to velocity

If a star's gravitational field causes a region to have a default expected Shapiro time-delay t, and the star has a relative velocity towards or away from us of v, we can expect to measure an additional velocity-dependent effect on the arrival time of a signal, of the order of $\pm vt/c$. [i]

The resulting (very artificial!) description of the problem then has light moving through the region normally, until is is absorbed by the point, and re-emitted later with the appropriate Shapiro time-delay: the distance that the point moves between absorption and re-emission then represents an advancement or retardation of the delayed signal, depending on whether the point (and its star) are moving in the same direction as the signal, or the opposite direction. The signal crosses the region more quickly if the star is moving in the same direction as the signals, and more slowly if the star moves in the opposite direction. The motion of the star has an effect on the effective one-way velocity of light across the region, that can be characterised as a "drag". [ii]

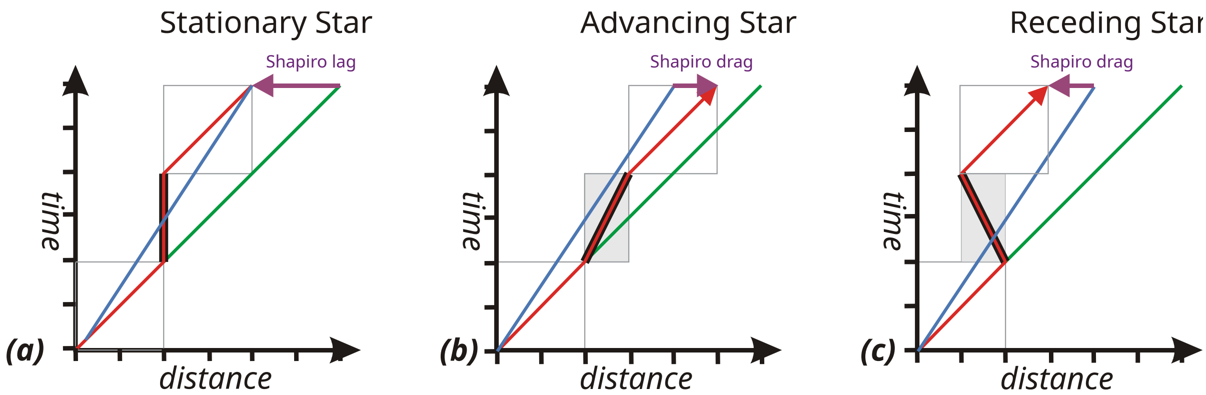

Figure 1: ***(a)*** *The usual path of light (red, bottom-left) is intercepted by a stationary transponder (black-outlined red), and re-emitted later, giving a timelag. The original expected spacetime path (green) can be replaced by the slowed averaged path (blue).* ***(b)*** *If the transponder advances in the same direction as the signal, there is a forward displacement and a reduced timelag.* ***(c)*** *If the transponder moves in the opposite direction, the signal gets a rearward spatial offset and a greater total timelag (first approximation).*

There is then an asymmetry in the one-way velocity of light measured across the region, depending on the direction of the beam, which can be represented as a "tipping over" of Minkowski lightcones. [21] This is, of course, just another way of describing the existence of gravitomagnetic frame-dragging effects due to the star's moving gravitational field.

Lightcone-tipping diagrams expressing the simpler light-velocity asymmetry due to the star's standard gravitational field can be found in Wald (1984) figure 6.12 [21] , and showing the rotational drag offset in Thorne (1987). [22]

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i This assumes the most simplified case of the star and its field modelled as a single transponder. It is a crude and unsophisticated initial calculation, vt being the distance moved during the Shapiro timelag, and division by c giving the time taken for light to cross the extra distance.
However, for the purposes of this paper, a qualitative calculation is sufficient.

ii While some might object to the use of the term "drag", the word seems functionally appropriate. While we *could* instead refer to it as a "*positive velocity-dependent proximity-dependent field-mediated momentum exchange effect*", it's not obvious how this would differ physically from calling it a "drag". The word "drag" is also already in widespread modern use with reference to accelerational frame-dragging, and rotational frame-dragging (which has a clear velocity component).

3. Wavelength-dependency

Ideally, if we use wavelengths of light to map out the light-geometry of a "*flat*" (or arbitrarily "smooth") region, it should not matter which wavelengths we use for the mapping. [i] However, if the region has warpage due to embedded particles and their fields, the distances reported can be expected to vary as a function of the wavelengths used to measure them. The basic principle is similar to the idea that a rowing-boat takes longer to cross the Atlantic than an ocean liner moving through water at the same nominal speed: the smaller boat has to navigate additional distances due to the curvature of waves and swells whose wavelengths that are shorter than the length of the liner's hull, and that the liner is oblivious to (*e.g.* Mandelbrot, 1967 **[24]**). [ii]

Applied to the Shapiro effect, we could say that, if the smallest feature of the Sun's exterior field corresponds to the Sun's own diameter (~1,391,400 km), then wavelengths smaller than this will be able to interact with the star's full curvature and will show the full Shapiro effect, but longer wavelengths will not be able to probe all the way to the bottom of the star's gravity-well and will show less of a Shapiro timelag (and also, correspondingly less of a gravitomagnetic drag).

Since optical frequencies of light are in the ~400-700 THz range (~400-700×10^{12} Hz), and EM gravitational waves with wavelengths of over ~1,391,400 km correspond to signals with frequencies around 0.21 Hz , it seems unlikely that the wavelength-dependency effect can be tested for with current equipment. [iii] [iv]

However, the effect *should* exist.

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i … as long as they are suitably smaller than the region to be measured! And not of *such* small wavelength that their energy-concentration physically warps the shape of the metric being assessed.

ii *See: "How long is the coastline of Great Britain?"* (**Mandelbrot** 1967), **[24]** and the scale-dependent measurements of lengths of fractal curves. A surveyor's wheel will not measure distances within surface features that are too small for the wheel to enter.

iii There are obvious problems constructing real or virtual reception and broadcast antennae over a million kilometres in length.

iv With a sine-wave signal, the longer the wavelength, the more difficult it is to accurately establish the exact moment of the wavelength peak, or the timing of the signal's zero-crossing.

4. Significance

4.1 Analogy with optical effects in refractive media

As previously mentioned, Irwin Shapiro was inspired to suggest the timelag effect by an analogy between nominal variations in the speed of light due to gravity (as in Einstein's 1911 paper [6]), and variations in the speed of light due to refractive index in particulate media.

What if the same laws apply to both? What are the expected results of assigning gravitational fields to the fundamental massed particles in a particulate medium?

- **Refractive index** – if every massed particle has a gravitational field, then light passing through a concentration of particles should take longer to reach the other side, due to the cumulative Shapiro timelag effects. The denser the packing of particles, the stronger the effect.
- **Dispersion** – for wavelengths longer than the characteristic scale of the particle curvature (say, greater than a particle's tiny calculated r=2M G/c^2 horizon radius), the speed of light in the particulate medium will be wavelength-dependent. Shorter-wavelength "blue" light will interact with more small-scale curvature and take longer to traverse a glass block than longer-wavelength "red" light. When a "white" mixture of wavelengths is passed through a glass prism, and crosses the air-glass boundaries at an angle, the deflection of light caused by the variation in lightspeeds will be different for different colours of light for whom the effective speed of light in glass is different.
- **Fresnel/Fizeau drag** – if the particulate medium moves, then the speed of light will be faster in the direction of the moving medium, than in the opposite direction.

These sorts of arguments were, of course, previously suggested in the context of variable-density aether theories of gravity … and rejected on the grounds that every different colour of light would then need its own physical aether. Geometrical physics is kinder in this regard: we can suggest a single agreed underlying metric that describes ***general reality***, with different-sized observers and different-sized sensors then generating different effective, *observable* geometries and metrics from the agreed, shared, underlying geometry.

As an abstract exercise, the phenomenology of particulate media – refractive index, Fizeau dragging and diffraction – can be described in terms of spacetime curvature. This makes it more difficult to argue that the realm of gravitational effects does *not* extend down into particle physics, as the effects of particulate media on light are at the very least *qualitatively* consistent with behaviour analogous to gravitational behaviours. [i] [ii]

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i A word-search of the Einstein Papers Project database at https://einsteinpapers.press.princeton.edu/ indicates that Einstein seems to have gone out of his way to avoid using the word "drag" in connection with the effects of general relativity. While he uses the word in the sense of "*aether drag*" and "*Fresnel drag*", and Fresnel "*drag coefficient*", the corresponding GR momentum-exchange effects for forcily-accelerated and rotating masses are instead referred to as "*gravitational induction*" (1912), or "*an inductive action of accelerated masses, of the same sign, upon the test body.*" (1921), or "*acceleration induction*" (1923).

Einstein's use of the word "induction" emphasised a correspondence to electromagnetic induction. However, since he'd previously presented the case for SR (in 1910 [25]) as being that special relativity was the only credible option left once we *dismissed* theories in which light was "dragged" theories, it may have been tactically inadvisable for him to suggest that, in the new context of general relativity, the dragging effects that he'd disparaged to arrive at the SR solution were real after all.

ii Einstein may not have realised that the relationships of special relativity were irreconcilable with the gravitomagnetic induction/dragging effects required for a general theory. According to a standard modern argument (MTW §6 [20]), the distortions and drag effects due to a body moving along an accelerated path are the aggregate of the distortions and drag effects associated with each of the velocity-stages that the accelerated path can be decomposed into. Since velocity-dependent distortion effects under SR have a strength of *zero*, we can then prove that their aggregate is *also* zero, and a forcibly-accelerated mass then *cannot possibly* warp spacetime, and must travel along a curved path through *flat* spacetime. [20] The logical conclusion is that if a general theory *requires* the existence of accelerational drag effects, it must also include underlying velocity-dependent drag effects, and therefore cannot be based on special relativity or Minkowski spacetime.

5. Context

5.1 Obviousness

Since the drag result seems to be *trivially correct*, one might wonder why it is so difficult to find any mention of it in textbooks and peer-reviewed papers. A mainstream physicist could argue that this is because *the result itself* is trivial, and too obvious to be worth publishing or discussing. According to this explanation, everyone (who has stopped to think about it) *already knows* that, for a region of slowed (or in the extremal case, *paused*) lightspeed, if the region moves, there should be an asymmetry in the rate at which light propagates through the larger containing region. [i]

5.2 Incompatibility

However, special relativity's motion-shift equations are founded on the critical assumption that the shape of spacetime is not affected by bodies' motions through it. A different explanation of the effect's obscurity is that, while the effect seems unavoidable according to the *gravitational* side of Einstein's theory, it clashes with the motion-shift relationships of special relativity.

An induced asymmetry in a region's light-velocities results in a deflection of lightbeams travelling through the region. A deflection counts as a change in the signal's *momentum*, with the momentum (and therefore also the received energy of the signal) increased in the direction of deflection. An onlooker will then see a *gravitomagnetic blueshift* if the signal is deflected towards them, and a *gravitomagnetic redshift* if the signal is deflected away.

This **gravitomagnetic velocity-dependent shift** in the energy and frequency of light is obviously not part of the SR flat-spacetime calculations of motion shifts, and cannot be retrofitted to the special theory, as accommodating the GM-modified wavelengths would mean allowing spacetime around a moving body to warp asymmetrically. Where there is a velocity-dependent drag, the region's lightbeam geometry is not fixed and Minkowskian, and the basic Doppler relationships cannot be those of special relativity.

5.3 Universal equations of motion

At this point a textbook adherent will argue, "Yes, but SR only claims correctness in cases where where gravitational effects are insignificant, and since any orderly deviation from SR lightbeam geometry counts as "gravitational" *by definition*, special relativity does not have to apply."

But relativity does not allow "strong-gravity" and "weak-gravity" masses to obey different equations of motion. If it *did*, then in the case of a neutron star moving relative to a grain of sand, we could compare the two sets of theoretical shift predictions with real-world data, in order to work out who was "really" moving, and how fast, wrecking the concept of relativity. For motion to be relative rather than absolute requires all massed particles, bodies, and systems to show *precisely the same* motion-shift relationships, meaning that there must be a single, *universal* set of equations that applies to all masses, everywhere. If special relativity's equation-set is exact for weak-gravity physics, it has to be just as exact for strong-gravity physics, and we cannot have gravitomagnetism. On the other hand, since a general theory of relativity *requires* gravitomagnetism, the universal motion-shift equations needed by generally-relativistic physics cannot be those of SR.

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i A similar question can be asked of the standard Shapiro effect: why did this have to be suggested by engineers rather than by professional theoretical physicists? (**Will:** " *… it is a mystery why Einstein did not discover this effect.* ", [4]
Carlip: " *In retrospect it could have been found much earlier.* " [26]).

6. Conclusions

Once we accept the principle of the Shapiro *timelag* effect, the Shapiro *drag* effect follows fairly inevitably. A moving gravity-source is then associated with an asymmetry in the one-way velocities of light through a region.

This amounts to a deflection of light at and around a moving gravitational body, in the direction of the body's motion, the effect being stronger with greater proximity. The resulting deflective field component is a **velocity-dependent gravitomagnetic field**, and can also be considered to be the body's **momentum field**, within which test particles undergo a deflection *via* **momentum exchange**. [i] The momentum field (the fieldlike distribution of the moving particle's *momentum* associated with the similar fieldlike distribution of its moving *mass*) meshes with the momentum *probability*-field under quantum mechanics, and can be used to model refractive index, dispersion and Fizeau drag, classically. It also underlies the rotational and accelerative "gravitational induction" fields [27] required for a general theory of relativity.

What makes the Shapiro drag effect politically awkward is that *any form* of velocity-dependent drag automatically invalidates special relativity. Einstein's special theory is built on the assumption that the simple relative motion of matter has no effect at all on the geometry of spacetime, and uses an assumed *fixed* (Minkowski) spacetime geometry [27] to derive the laws of physics. If spacetime geometry changes *in any way* as a function of a body's relative velocity, then our equations of motion and Doppler relationships must be non-Lorentz [ii] [iii] … everywhere.

The fact that a "Shapiro drag" invalidates Einstein's system may explain why there seems to be little or no mention of it in the peer-reviewed literature. This may also explain the mystery of why it took so long to document the *standard* Shapiro effect, and why the "discovery" paper came from someone outside the main GR community. It is conceivable that Einstein might have already been aware of this class of argument when he presented the 1916 version of his general theory, but may have preferred not to mention it due to its toxicity to the framework that he was trying to promote at the time. Other experts may have then chosen not to explore the subject on the grounds that there seemed to be no way to get the consequences (velocity-dragging) to mesh with Einstein's system, leaving the field open for Shapiro.

If moving bodies *can* drag light, the appropriate choice of relativistic model would be a dynamic-geometry Hertzian system of relativity, [25] [28] rather than the fixed-geometry Lorentzian system that Einstein adopted in 1905, and then went on to use as the foundation of his general theory. [29] [iv] If the Shapiro drag is *real*, it would seem that Einstein based his general theory on the wrong model.

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i A collision with a moving mass-field being equivalent to a *partial* collision with the mass *via* its field, which acts as a proxy.

ii The relativity principle only allows a single, universal set of basic equations of motion that must apply to all simple moving bodies regardless of their compositions and properties. If a moving neutron star shows Shapiro drag, and the drag changes the momentum and energy of light, the received frequencies of light-signals cannot agree exactly with the SR predictions. But the requirement of universality means that the same strong gravitomagnetic deviation from SR's predictions must then hold exactly *for all moving bodies whatsoever*, right down to the scale of the smallest possible fundamental massed particles. Fundamental massed particles are condemned to either show precisely the same gravitomagnetic deviation as a moving neutron star, or an identical deviation due to a conspiracy of other factors.

iii The Principle of Equivalence of inertia and gravitation insists that all *inertial* masses are also *gravitational* masses. If SR does not apply to *gravitational* masses, then it does not apply to *any* masses (except in abstract geometrical exercises).

iv SR can be characterised as "relativity plus flat spacetime" — based on the belief that the presence and motion of matter has no effect at all on lightbeam geometry "*We know that moving matter does not drag light*". SR's interpretation of the lightspeed-constancy postulate interprets it to mean *global* lightspeed-constancy throughout a region, and not just "local c" for individual observers. Observer-masses are then required *not* to have their own gravitational or gravitomagnetic fields. [30] If this is wrong, and simply-moving matter *can* drag light, relativistic physics defaults to a different form.

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