

Shift-symmetry in Einstein's universe:

Part D: Cosmology

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Part of a series. This paper addresses the incompatibilities between modern cosmology and Einstein's assumed symmetries of Doppler shifts, gravitational shifts and timeflow.

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doi: 10.13140/RG.2.2.11760.06401

1 Introduction

This is the fifth of a series of papers [\[1\]](#) [\[2\]](#) [\[3\]](#) [\[4\]](#) [\[5\]](#) [\[6\]](#) exploring Einstein's concept of shift-symmetry.

The introductory paper gave an overview of the shift symmetry principle implicit in Einstein's work, and the three mutually-supporting "pillars" of his system of thought. [\[1\]](#) **Paper A** then showed that gravitomagnetism, essential for the implementation of the general principle of relativity, cannot exist in a universe that supports special relativity's equations. [\[2\]](#) **Paper B** showed that gravitational shift symmetry leads to an inconsistent classical theory of gravity, and generates absolute event horizons, which are incompatible with quantum mechanics. [\[3\]](#) **Paper C** argued that gravitomagnetism's equations must be asymmetrical with respect to time, and that gravitomagnetic equations require a thermal system to constantly be losing energy through the emission of g-waves. [\[4\]](#) [\[7\]](#) [\[8\]](#) [\[9\]](#) [\[10\]](#)

Having dealt with (and disposed of) Einstein's "three pillars", this paper addresses a fourth subject that was not yet fully developed in 1915/1916, the geometries associated with cosmology, specifically, those for a Hubble universe. Once again, we will find that Einstein has not derived the new results from his earlier theory or equations, but has developed new arguments for the new context and attached them to the existing theory, without validating compatibility. Checking, we find that we cannot model the critical relationships of a modern "Hubble" universe correctly if special relativity's symmetrical equations are assumed to be correct.

This paper presents a mixture of standard and nonstandard cosmologies for comparison.

2 Symmetrical and asymmetrical systems and cosmologies

2.1 The fully-symmetrical Einstein system (1915/1916)

We will start (sections 2.1 and 2.2) by quickly revisiting some of the findings of the earlier papers in this series.

Einstein's system assumes symmetry. [1] With the Lorentz-Einstein recession Doppler relationship, $E'/E = \sqrt{(c-v)/(c+v)}$, [11] a simple collinear motion-shift or gravitational shift inverts when we invert the velocity's polarity. [2] The resulting equations for physics are perfectly time-symmetrical, with a gravitational redshift becoming a gravitational blueshift under time-reversal. [3] With Einstein's foundation equations there is no arrow of time either locally or globally. [12] [4]

Because two successive shifts of the same magnitude but opposite polarity cancel perfectly, this gives a perfect implementation of traditional energy-conservation, ⁱ and a signal that passes through a gravity-well returns to its original height with exactly the same energy that it started out with. [3] ⁱⁱ When the signal crosses cosmologically-significant distances, riding in and out of a great number of gravity-wells, there is then no cumulative effect on the signal.

If spatial or spacetime curvature is to be evaluated by its effect on light, then Einstein's symmetrical SR shift equations generate a cosmology that has no cumulative effects with distance: it is flat, **pseudo-Euclidean**, ⁱⁱⁱ infinite in spatial and temporal extent, and (due to time-symmetry) neither expanding or contracting.

i .., until we factor in the energy-requirements of gravitational waves, at which point the Einstein system breaks down. [2]

ii By contrast, in a gravitomagnetic model, the signal returns to its original height with an energy-loss.

iii **Pseudo-Euclidean geometry** can be illustrated by the shape of the cardboard egg-trays that one sees in farmer's markets. The egg-tray has lots of obvious regions of positive curvature, the convex/concave "cup" regions that hold the eggs safe. However, the tray has no cumulative curvature, and manages to lie flat on a table without rolling up into a ball, because it also has lots of regions of negative curvature ("saddle-shaped" or "Pringle-shaped" regions).

A "pseudo-Euclidean" region can appear flat at first sight, but as we investigate further, we find positive curvature at smaller scales, which is balanced and cancelled overall by matching negative curvatures.

2.2 Asymmetrical systems

2.2.1 Asymmetrical behaviours

On the other hand, the general principle of relativity – the relativity of acceleration and rotation – requires both **accelerative and rotational gravitomagnetism**, which in turn require an underlying *velocity-dependent* gravitomagnetic effect that is geometrically incompatible with SR. ⁱ Inertial physics under a gravitomagnetic theory then has to be described by a dynamic, relativistic acoustic metric ^[14] rather than the fixed metric of Minkowski spacetime, ^[15] and with the transition from fixed to dynamic geometry come some non-Minkowski characteristics, most notably in the form of different Doppler relationships.

All relativistic solutions diverge from their neighbours by “Lorentzlike” factors, and since any Lorentzlike deviation from the SR solution breaks symmetry, ⁱⁱ a gravitomagnetic theory’s relationships, which must be *non-SR* as a matter of geometry, are obliged to be *asymmetrical*. ^{[16] [17]}

Relativistic gravitation ^[18] requires the resulting asymmetrical Doppler relationships to diverge from (and be redder than) their SR counterparts. ⁱⁱⁱ Gravitational shifts are then path-dependent, and a signal aimed through a gravitational feature arrives back at its original height with a round-trip energy-loss. ^{iv v [19]}

2.2.2 Cosmology in an asymmetrical universe

We now have enough background knowledge to consider cosmological questions.

For signals sent over vast cosmological distances, assuming a broadly uniform distribution of clumpy matter, we should expect a cumulative residual redshift as the signal rides in and out of the gravitational features that it encounters *en route*, roughly as a function of distance travelled.

This cosmological redshift prediction is not an “alternative explanation” of Hubble effects that eliminates the need for expansion ... it is, rather, *an alternative method of deriving expansion*. ^{vi vii}

i Alternatively, if matter has rest curvature, and the speed of gravity is finite [13], moving matter must have an associated *gravitomagnetic* curvature, and the shape of a region of spacetime must change as we hurl bodies through it with different velocities.

ii SR’s predicted recession redshifts and approach blueshifts for a given velocity cancel when they are multiplied together, giving shift-symmetry). However, even the smallest Lorentzlike deviation from SR *breaks* this property, as a Lorentzlike deviation (to the blue or red) would have to apply in the same way to both redshifts and blueshifts, giving a residual *deviation-squared* after multiplication.

Since the SR equations make gravitomagnetism impossible, and any modification of the equations to allow GM effects destroys symmetry, a general theory of relativity cannot be shift-symmetrical.

iii It is more correct to say that a general theory requires the equations to be *different* to SR’s by *some* form of Lorentzlike factor, either to the red or to the blue. Whichever we choose, the opposite applies in reversed time. A system then gains energy in one time-direction, and loses it in the other. We can then invoke the “*no infinite energy machines*” rule establish a convention that “forward time” is defined as being whichever time direction is associated with systems losing energy. [7]

The gravitomagnetic equations required for a working general theory of relativity must differ from the SR version, to support the velocity-dependent curvature effects that are absent in SR. Hence, we can say that the correct general relativistic equations are necessarily redder than those of SR ... in forward time.

iv The problem of “no infinite energy machines” in *reversed* time is then resolved by saying that if reversed time cannot be experienced, its descriptions are not descriptions of physical reality. [7] If an observer experiencing reversed time would *already* be violating physical law, then physical law cannot be expected to provide a consistent description of their (impossible) experiences.

v Since Hubble redshifts *already* break traditional energy-conservation laws, the subject needs to be revisited anyway.

vi Under a geometrically-consistent theory of physics, a single shared shift law then applies to motion, to gravitation and to cosmological expansion, allowing these “different” effects to be interchangeable

vii In a gravitomagnetic model, a Doppler shift due to the simple relative velocity of a body can be recalculated instead as the result of velocity-dependent gravitomagnetic curvature along the signal path.

As a result:

- **Reinterpreted as a *motion shift***,ⁱ the asymmetrical distance-dependent redshift will tell us that more distant objects are receding faster, and that the whole universe is expanding, or,
- **Reinterpreted as a *gravitational shift***, the predicted distance-dependence of the redshift tells us that the further away a body is – and the longer ago the events that we are witnessing – the denser the region of spacetime was at the moment of emission. Field-density as a function of *time* suggests that the universe must have been more compact in the past, and that it is (again) steadily expanding.

Either way, it is the *asymmetrical* shift equations (not Einstein’s symmetrical set) that predict a distance-dependent redshift and an expanding universe, and that can claim “ownership” of the Hubble redshift predictions,.

If we had been able to conduct a proper analysis of Einstein’s 1916 theory at the time, by evaluating whether or not the basic equations of Nature should be symmetrical, we could have said that the *experimentum crucis* for Einstein’s 1916 theory would be the *absence* of distance-dependent redshifts. Edwin Hubble’s subsequent 1929 redshift results [22] would then have been seen as strong experimental evidence against Einstein’s system.

2.3 Einstein’s first hybrid system (1917)

In 1917, Einstein published an auxiliary paper on cosmology accepting that the universe was probably spatially (hyper)spherical. [23] [24] If the universe was *infinite and unbounded*, with an even distribution of matter, and infinitely old, it would have an infinite field-density everywhere. But one could engineer a metric whose density petered out with distance from a central point, and populate it with a finite amount of matter, distributed in sympathy with the metric, in such a way that to inhabitants it seemed finite and everywhere the same. [21]

Friedman (1922): [25] “Einstein obtains the so-called cylindrical world, in which space possesses a constant curvature independent of time.”

While this was an improvement on the 1915/1916 description, the “gravity is cumulative” arguments for a spherical universe did not obviously mesh with Einstein’s own equations.

Einstein, wanting the universe to be of fixed size (and time-symmetrical) [1] effectively eliminated this cumulative redshift and restored gravitational shift-symmetry [3] by inventing a special repulsive effect that he called the **Cosmological Constant** (“**Lambda**”, Λ), [23] [26] which would have generated an exactly-cancelling distance-dependent *blueshift*.

What wasn’t clear was why a universe with Lambda, which cancelled the *physical evidence* of sphericity, and cumulative curvature, still somehow managed to be spherical. ⁱⁱ By getting rid of distance-dependent redshifts, Lambda brought the universe’s large-scale behaviour back into line with the 1905/1916 equations, but the absence of distance-dependent effects then made it difficult to argue that this universe had any cumulative positive curvature that caused it to be curled up into a ball. ⁱⁱⁱ

i ... using a non-SR shift equation.

ii If gravitational (positive curvature) effects are cumulative, a 1916 pseudo-Euclidean universe requires a cancelling source of negative curvature, and this is supplied by the 1917 Lambda.

iii Einstein wasn’t alone in adding apparently whimsical elements to his cosmological model, without supporting derivation or analysis. His correspondence with other cosmologists, post-1916 shows a surprising amount of unsupported assertion, argument from personal conviction, and sometimes a surprising lack of knowledge of previous work, from all parties. (e.g. [27])

The concept of a positively-curved universe was not new: a centenary review paper [24] notes at least *nine* major researchers as already having explored the idea and consequences of a spherical universe by the end of 1900. ⁱ Einstein was selectively grafting these arguments onto the physics, and claiming the results for his general theory, without appearing to worry too much whether the predicted outcomes were in agreement. ⁱⁱ ⁱⁱⁱ

Since Einstein's approach did not include a link between a necessary hyperspherical curvature and the actions of matter on light necessary to *produce* such a curvature, deSitter was able to produce a solution of Einstein's equations in which spacetime was still curved in on itself despite the solution's universe containing no matter at all! [28] DeSitter's publication did not amuse Einstein, as it could be taken as demonstrating that Einstein's construction was artificial, and that the complete dependency of universe's spacetime geometry on its contents, that he claimed for his theory, was not actually present in the gravitational equations. ^{iv}

2.4 Einstein's second hybrid system (~1930/1931)

After initially rejecting Friedman's dynamic, variable-size universe [31] [32] as faulty, [34] Einstein admitted in 1923 that he'd made a mistake. [35] But he still didn't seem to fully accept expanding-universe cosmology as the right answer until the early 1930s. [36] [37] ^v It took some time for Einstein to accept the Friedman and Lemaitre [42] arguments for the "*abominable*" (!!!) [36] expansion and the Hubble result. Accepting that he had missed the opportunity to predict Hubble redshifts by just a few years, and that Lambda should be assigned the value "zero", Einstein reportedly told Gamow that this had been the biggest blunder of his career. [43]

With the adoption of external arguments from outside researchers, Einstein's third cosmology was (hyper)spherical and expanding, with a Hubble redshift in accordance with the recent experimental data. However, although the supporting arguments for this cosmology were strong, they were not obviously Einstein's, and it was again not clear that their geometry meshed with the equations of Einstein's own theory.

After all ... it had been the SR/Schwarzschild relationships that had let us prove the *absence* of distance-dependent redshifts, and if we were to attempt to map out the geometry of a region by mapping the properties of signals sent through it, the total cancellation of gravitational shifts in the SR/Schwarzschild description would have suggested that a region populated by gravity-sources should show *no* cumulative curvature. ^{vi}

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- i O'Raifeartaigh, O'Keeffe, Nahm, and Mitton (2017) [24] mention the earlier research into non-Euclidean cosmologies of Lobachevsky (1856), Riemann (1873), Zöllner (1872), Newcomb (1898); Peirce (1891), Ball (1881) and Schwarzschild (1900), along with Gauss, and Bolyai. This list is probably not exhaustive.
 - ii Riemann's 1924 solution with negative curvature [33] demonstrated that Einstein's 1917 preference for a spatially closed universe with positive curvature did not come from his gravitational equations. It required further (or other) assumptions.
 - iii This can be compared to Einstein's similarly "loose" approach to general relativity, in which he "retrofits" general relativity to the existing structure of special relativity without checking for compatibility, while acknowledging (and arbitrarily rejecting!) the logical possibility that his presented reduction of GR to SR might not be physically meaningful. [29]
 - iv Which is in a sense unsurprising, as the underlying SR equations, derived for physics in flat spacetime, depend on there being *no* dependency of the shape of spacetime on the presence or absence of matter.
 - v Einstein also published yet another cosmology, hyperspherical with regard to *spacetime* [38] (*i.e.* the Hawking-Hartle "bubble" [40]). This was a concept that he'd dismissed when deSitter had presented it to him, and was arguably anticipated by Poe. [41]
 - vi Another interpretation might be that the SR equations are simply not compatible with a universe that supports gravitation.

3 Cosmological horizons are non-Einstein

3.1 Cosmological horizons are relative horizons

A cosmological expansion horizon is not like the absolute horizons described by special relativity's equations and the Schwarzschild solution, with a location agreed upon by all observers [3] ... it is instead an observer-dependent, *relative* horizon – with different onlookers projecting their own cosmological horizons onto different locations.

Relative horizons have some superficial similarities with conventional planetary observation horizons, in that they are assigned to different locations as a function of the positions and properties of the observers for whom they are calculated. Different observers are entitled to disagree as to where a cosmological horizon, projected from their own different locations, and calculated using their own different gravitational properties, ought to be. ⁱ

Relative horizons show the same broad behaviours (path-dependence, observer-dependence, mass-energy-information leakage along accelerated paths) as the gravitational horizons generated by an *acoustic metric*. [14] They also support indirect-transmission mechanisms that correspond to “Hawking radiation” effects under QM, with the background temperature of a cosmological horizon corresponding to the Hawking temperature. ⁱⁱ [44]

3.2 Relative horizons do not obey SR shift laws

The application of the symmetrical SR/GR1916 shift relationships [1] requires all resulting horizons to be absolute event horizons, ⁱⁱⁱ [3] Since cosmological horizons *cannot* be event horizons, their behaviour cannot correspond to the shift characteristics of SR/GR1916.

3.3 Cosmological horizons are non-SR physics

It is now generally accepted by most of the cosmology community that Hubble redshifts cannot be calculated properly by taking the expected recession speed of a star or galaxy and applying the SR recession redshift formula.

This is not least because the SR formula generates a velocity-addition formula that applies regardless of whether matter exists in a region or not, whereas the acoustic metric arguments that apply when we have relative horizons require a different form of velocity-addition formula, whose application depends on the velocity-stages being marked by real objects, and supports trans-horizon behaviours that can assign a signal horizon to different positions depending on the properties of the population of masses affecting the signal path.

Cosmological, *relative* horizons are natural features of a *dynamic* metric rather than the fixed Minkowski metric, and since the Minkowski metric is an expression of the SR equation-set, relative cosmological horizons are not SR-compatible. ^{iv}

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- i We know that relative horizons are not event horizons, because a surface that corresponds to a horizon for one observer is a region of normal space for another, allowing information-flow in all directions.
 - ii If we had developed a “gravitomagnetic” general theory based on purely-local c and Hertz’ 1890 light-dragging concept [45] rather than on Lorentz and Einstein’s “global c ”, Hubble cosmology’s relative cosmological horizons could have been seen as a natural extension of the theory’s relative gravitational horizons. [46]
However, this correspondence does not exist with Einstein’s system.
 - iii ... because an observer at null infinity sees the shift at a horizon to be $E'/E=0$, the return blueshift needs to be $E'/E = (1/0) = \infty$. With the SR equations, a horizon surface is a “special” location, at which objects tend to infall at or around the background speed of light, being stationary with respect to the surrounding stars is illegal, and at which mass-energy and information can only flow inwards, not outwards. Since *any point in space* can be argued to lie on a cosmological horizon for *some* future distant onlooker, *cosmological* horizons cannot be “special” locations in this sense.
 - iv Although Einstein agreed that switching from SR to GR should be accompanied by a change away from an absolute metric [15] his retention of the SR equations as a foundation led to the retention of some default non-dynamic characteristics.

4 Are cosmological shifts Doppler shifts?

4.1 Einstein says, “Yes”

The incompatibility between Einstein’s equations and cosmological horizons is troublesome, as the “geometrical” and “local physics” approaches ... where we calculate the effect of light only on the geometry that a signal comes into direct contact with ... needs to be able to calculate shifts purely from the geometry along a signal path, without knowing or caring about the reason why that geometry exists.

Einstein, confident that his system was correct, was happy to announce that a Hubble recession shift *could* be treated as a conventional motion-shift effect.

Einstein (1920): [48] “ [Hubble shift] can be interpreted with regard to our present knowledge only in the sense of Doppler’s principle, as an expansive motion of the system of stars in the large ”

Einstein (1946): [49] “ Some try to explain Hubble’s shift of spectral lines by means other than the Doppler effect. There is, however, no support for such a conception in the known physical facts. ... This would mean that there exists no metric in the sense of relativity. ... it may be that some proponents of the above hypothesis imagine that the process of the expansion of light is not according to wave theory ... ”

But the Doppler effect (as revealed by cosmology) follows a different relationship to that of Einstein’s 1905 and 1916 theories.

4.2 Other researchers say: “Not quite”

If the basic shift equations of Einstein’s special and general theories are correct, Hubble shifts **cannot** be described as Doppler shifts, at least, not as *Einstein-compatible* Doppler shifts.

In Rindler’s more pragmatic (and possibly more informed) worldview, if we were working strictly within Einsteinian relativity, where the characteristics of Doppler effects were defined by special relativity, cosmological shifts could not properly be referred to as motion shifts, and had to be called Something Else.

Rindler (1969): [50] “ Note that the cosmological redshift is really an expansion effect rather than a velocity effect ”

We *had* to convince ourselves that there was some sort of special distinction between the two classes of recession redshift, and find a way to rationalise this position, otherwise the two most important classical theories of the early C20th would be invalidated.

Seeds (1992): [51] “ Although astronomers and cosmologists commonly refer to these red shifts as Doppler shifts, and often speak of the recession of the galaxies ... The expansion of the universe is an expansion of the geometry and not just a simple recession of the galaxies. ”

The point of these qualifications is to establish the principle that Hubble shifts have “special permission” not to conform to the SR Doppler predictions. Declaring that these two geometrical recession-shift effects as “different things” that are *supposed* to follow different equations gives us a defence against accusations that GR1916’s geometrical description is inconsistent with modern cosmology. ⁱ

i ... so the GR1916 sceptic will complain “*This doesn’t agree*”, and the Einstein apologist will counter “*It’s not supposed to*”.

4.3 Rationalisations

Some commentators argue that

*“Special relativity is a **flat**-spacetime problem and cosmological shifts are a **curved**-spacetime problem, so **of course** the equations are different!”*

This argument contains at least an element of geometrical truth. If we have two different scenarios, one where relative velocity *must always* be associated with curvature, and another where it *must never* be associated with curvature, the geometry-change as a function of velocity in the first case implies that the two descriptions cannot use the same geometrical description or equations.

But this line of reasoning *also invalidates Einstein’s 1916 theory*. GR1916 takes the SR equations derived for velocity-differentials in *flat* spacetime, and generates gravitational shift predictions by applying the same equations to velocity-differentials in *curved* spacetime, to give the Schwarzschild solution. If velocity-effects that do and do not associate a velocity-differential with curvature should not use the same equations, then Einstein’s 1916 system is already a wrong theory. The argument already rules out Einstein’s SR-centric architecture and before we even get as far as cosmology. ⁱ

For sufficiently-small values of v/c , the inappropriateness of the Einstein Doppler equations isn’t detectable. Hence, the interesting mix of provisionals and absolutes in **Shutz (2009)**: ^[52] “... *provided the galaxy is not far away. In our cosmological neighborhood, ... the cosmological redshift is a true Doppler shift.*” So the effect *is* a fundamental Doppler shift, unless v/c is large enough to reveal that its equations disagree with SR, at which point it isn’t. (?!?)

Another attempt to reconcile the irreconcilable appears in **Wald (1984)**: ^[53] “*Note that v can be greater than the speed of light if R is large enough. This does not contradict the fundamental tenet of special and general relativity that ‘nothing can travel faster than the speed of light’, since this tenet refers to the locally measured relative velocity of two objects at the same spacetime event, not a globally defined velocity between distant objects.*”

Einstein’s 1905 velocity definitions *were* global, and referenced to frames, not events (events were used by SR to reference distant simultaneity). Wald’s willingness to change the basic definitions of special relativity to argue the absence of a problem illustrates the problem’s intractability.

i How do we fix this aspect of the 1916 theory?

- Gravitation is clearly a curved-spacetime problem, so the association between velocity-differential and curvature under a general theory is non-negotiable.
- Similarly non-negotiable is the fact that the shift on a signal falling across a gravitational gradient needs to be the same as the motion shift on a body falling across the same gradient.
- So, in the context of a general theory, if gravitational shifts cannot be described correctly by flat-spacetime equations, then neither can conventional motion shifts. Simple relative velocity between masses must then always be associated with curvature ... as in fact they would be in a gravitomagnetic theory of inertial motion derived for a universe conforming with the GPoR.

With the “new” Doppler equations derived for curved spacetime, cosmological and inertial velocity-differentials are both associated with (velocity-equivalent) curvature along the signal path. We then can describe Hubble shifts as simple recession shifts, or as simple gravitational shifts. The caveat here is that the “universal” set of shift equations, which then describes motion shifts, gravitational shifts and cosmological shifts, is not Einstein’s.

5 A universe cannot support multiple shift relationships

5.1 Incompatibility with topology in Einstein's system

A geometrical theory must still work in exactly the same way when its geometry and laws are subjected to geometrical transforms. Since a transformation merely gives a different view of the same data and the same interrelations – the same universe projected onto a different surface or coordinate system – our final physical laws are still obliged to work regardless of how we distort their descriptions.

One such topological transformation involves “turning the universe inside out”. ⁱ [54] Instead of the Earth's surface facing outwards and being surrounded by a cosmological curvature horizon, we can take the Earth's surface as an *inward*-looking surface, surrounding the universe's contents. This exercise turns the original surrounding cosmological curvature and horizon into a central contained gravity-well, with a curvature-horizon, that then has to obey the same exterior geometrical rules as the other contained (gravitational) curvature-horizons. If we select the gravitational horizon of a collapsed star as the starting-point for our exercise, and have *that* described as the universe's surrounding inward-facing limit, then the visible region around this horizon also takes on the identity (in the description) of the region inside a surrounding cosmological horizon.

The same physical laws and geometrical rules must then apply to both cosmological and gravitational curvature. If cosmological horizons are observer-dependent, “acoustic”, and described using non-SR shift equations, then the same must apply to the horizon physics of a collapsed star.

Expanding-universe cosmology plus topology tells us that gravitational horizons cannot be event horizons, that the Schwarzschild solution cannot be valid, that Einstein's general theory must be wrong for adopting the symmetrical SR relationships.

5.2 ... but expansion shifts are also gravitational shifts!

The previous topological arguments may lie outside our comfort zone, and we may not feel sufficiently familiar with them to be able to confidently judge their validity.

However, in a very real sense, Hubble redshifts **REALLY ARE** gravitational redshifts, of an unconventional type. [55] When a signal spends millions or billions of years in flight, the ongoing expansion of the universe causes the region in which it is finally detected to be a more rarefied gravitational environment than the region in the earlier, denser universe in which the signal was generated. There is therefore a **gravitational field-density differential** between the source and destination regions of spacetime – the signal is effectively climbing an “uphill gradient” between a denser (earlier) and a less-dense (later) region, and therefore needs to arrive with a gravitational redshift.

If the cosmological shift can be expressed as a gravitational shift due to a density-differential along the path, then the previous arguments apply: gravitational shifts must have the same non-SR equations and behaviours as cosmological shifts, the SR-centric Schwarzschild solution has to be wrong, and the required linkage between gravitational and inertial physics equations then also eliminates special relativity.

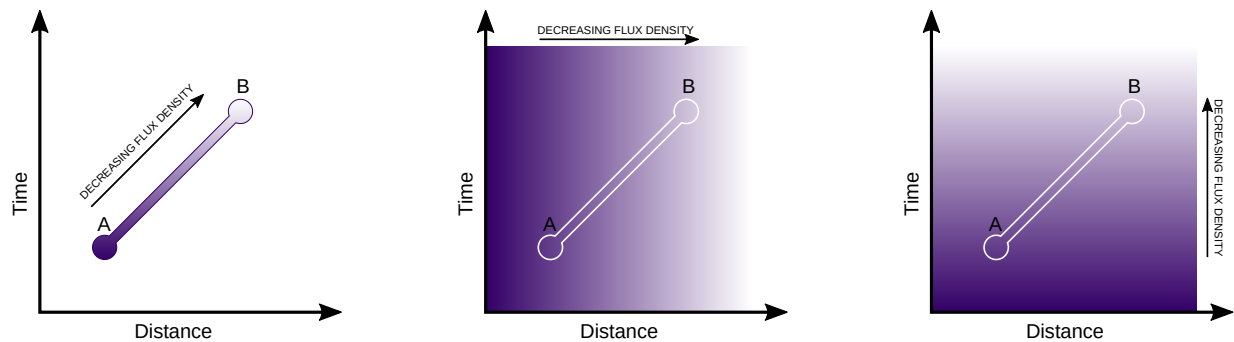
ⁱ See the “*Farside Black Hole*” exercise in **Relativity in Curved Spacetime**, chapter 12.13. [54]

5.3 Orientation of gravitational and cosmological fields in spacetime

5.3.1 Base definitions

Regarding cosmological and gravitational shifts, we can say that:

- **A conventional gravitational shift** is caused by a density-differential between two regions of *space* that a signal moves between (the differential and gradient are “spatially-aligned”).
- **A Hubble redshift** is caused by a density-differential between two periods of *time* that a signal moves between (the differential and gradient are “temporally-aligned”).



Identical density-differentials along a signal path, (i), created by (ii) a gravitational field and (iii) by cosmological expansion [55]

We can then say that the difference between a gravitational shift and a cosmological shift is the spacetime alignment of its density-differential. If the density-differential is aligned only with the space coordinates, and is constant over time, then we have a conventional static *gravitational* field whose strength varies from place to place. On the other hand if we have a differential that is the same everywhere in space, but varies with *time*, we have a *cosmological* field.

The difficulty faced by Einstein’s system, which requires both sets of effects to be different, and to follow different equations, is that the path of a signal in real life makes a diagonal across *both* space and time, so that, by only looking at the density-differentials along the signal path, we cannot immediately tell whether the supposed cause of the differential is cosmological or gravitational. [55]

The principle of local physics says that a system or region’s internal data should be enough to predict the behaviour of that system or region without reference to anything external. [i](#) [ii](#) [iii](#) [iv](#)

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- i ... until “unknowns” have propagated inwards from outside the system sufficiently deeply to affect the items under study.
 - ii This introduces the assumption that the shift is calculable from just the differential and is unaffected by the angles of vector properties crossing its path. But Einstein’s system makes the same assumption. If crossing-vectors are an issue, we need to invent a new physics anyway.
 - iii The considerations of **Paper A** [2] and **Paper B** [3] (Einstein’s SR-centric general theory is incompatible with gravitomagnetism and gravitation) already show that the system is already not a valid theory of classical physics, even *before* we consider cosmological effects. However, Hubble shifts throw up whole new categories of incompatibility.
 - iv **MTW page 4:** [56] “*Physics is simple only when analysed locally This is surely Einstein’s concept that all physics takes place by ‘local action.’*”

MTW page 19: “*In analyzing physics in a local inertial frame of reference, or following an ant on his little section of apple skin, one wins simplicity by foregoing every reference to what is far away. Physics is simple only when viewed locally: that is Einstein’s great lesson.*”

MTW Box 1.6: “*In contrast, it was the whole point of Einstein that physics looks simple only when analyzed locally.*”

Even if it was possible to distinguish between the effects of cosmological and gravitational curvatures on a signal, to do so would be to break the principle of locality.

If the shifts resulting along the path depend on properties *outside* the path, then we have non-local effects, and with nonlocality, we no longer have classical physics.

Discarding classical physics in order to try to save Einstein's special and general theories, results in the loss of the classical foundation that both theories are based on, leaving us with nothing. If we keep locality, then cosmological and gravitational shifts must follow the same law, and we lose Einstein's SR-centric system.

5.3.2 Relativistic cosmology: "Tilt" of coordinate axes with velocity

The earlier arguments show that, according to current mainstream theory, a given field density differential along a path needs to affect light differently depending on whether it is caused by the path intersecting a *spacelike* field-density-gradient (gravitational shift), a *timelike* field-density gradient (expansion shift), or some mixture of the two.

This situation gets worse when we allow bodies to have conventional relative motions, ⁱ as our sense of the orientation of space and time varies with our state of motion relative to other bodies (or relative to our own past states of motion).

- If we move away from a constant gravitational source, the environmental field-density at our location reduces with time, and the field-gradient no longer seems to be aligned purely with respect to *space*, and now (for us) has a time-aligned component, as if it was "cosmological".
- Now suppose that a cluster of stars and galaxies happens to have no significant relative motion other than that due to cosmological expansion, and that the group as a whole also has no net motion with respect to the local cosmological average. We will expect the cosmological expansion vector to be roughly parallel to the stars' worldlines, and the stars to experience the cosmological density-gradient as being time-aligned. But if we now *move though* this collection of stars at a high velocity, the expansion vector will be tilted at an angle to our new worldline-reference. The decreasing forward timelag on signals ahead of us makes it seem as if that region is ageing (and expanding) faster, and the increasing timelag on signals behind us makes it seem as if *that* region of universe is ageing (and expanding) more slowly. The cosmological variation in density now also appears to vary with spatial position, with the spatial component appearing to us as a *gravitational* shift component. ⁱⁱ

This "cross-contamination" of spatial and temporal gradients makes it more difficult to argue that these are distinctively separate effects, obeying different rules.

i Einstein dealt with the problem of relative motion in cosmology by deleting it from the description. His simplified field equations for cosmology assumed that matter had **no** relative motion, other than cosmological.

ii We might then expect the rearward, redshifted stars to pull more strongly than the forward blueshifted stars, causing a fast-moving object to slow towards the average state of motion of its surroundings. This coincides with a gravitomagnetic argument that we should be able to treat the recession and approach redshifts and blueshifts of stars as evidence of a real gravitomagnetic field.

In the gravitomagnetic case, the difference between the gravitational effects of blueshifted and redshifted stars is countered by the stars' apparent change in angular distribution, [57] so that the effect of the reduced pull of individual stars ahead of the observers as opposed to behind them is exactly matched by *increased numbers* of forward stars and *decreased* number of rearward stars. Newton's First Law ceases to be a requirement that we put in "by hand", and becomes an emergent effect within curved spacetime and curved-spacetime gravitomagnetic physics. [58]

6 Summary: An embarrassing multiplicity of rules

6.1 Preserving Einstein

The simplest interpretation of this situation is that Einstein's system cannot be extended to include Hubble shifts, because it has the wrong equations for our universe. Attempting to save Einstein requires *the multiplication of entities*, contrary to Occam's razor:

- **Cosmological recession redshifts are non-SR.** Protecting the SR shift equations from invalidation requires us to have two different versions of the recession redshift, and to distinguish between “conventional” and “cosmological” recession velocities, following different equations.
- **Cosmological redshifts are also calculable as gravitational redshifts.** But since gravitational shifts inherit the relationships of inertial physics, *Einstein's* “SR-compatible” [3] gravitational redshifts, which generate absolute horizons, are incompatible with Hubble shifts.
To calculate Hubble shifts as gravitational shifts without invalidating Einstein's system, we also now need two different versions of the *gravitational* shift equations.
- **Conventional gravitational shifts can have a time-variant component at the observer's location when there is relative velocity.** If we are saying that time-aligned cosmological shifts normally have non-Einstein laws, but space-aligned gravitational shifts follow GR1916, then when a GR1916 gravitational shift *acquires* a time-aligned component due to relative motion, for its relationships not to suddenly jump to the non-Einstein relationships used for Hubble shift, we would seem to need two versions of the time-aligned equations, as well. ⁱ

In order to avoid contradicting the equations of Einstein's system, we need to distinguish between three distinctly different velocity-shift effects, and then support two versions of each, cosmological and non-cosmological, giving six cases in total. There is no obvious way of reconciling these six different shift effects with a single geometry.

6.2 The gravitomagnetic theory alternative

On the other hand, if we had developed relativity theory around *purely local c*-constancy as suggested by Hertz, [45] [46] we would be able to describe a velocity shift either in the time domain as the result of changing separation, or in the space/spacetime domain as the result of an apparent gravitomagnetic differential. Motion shifts, gravitational shifts, and cosmological shifts would then *all* be describable as curvature shifts, and would all use the same non-SR equation-set. ⁱⁱ

Einstein's six different cases, with two equations, would collapse down to just one, *universal* case, and a single equation. ⁱⁱⁱ

i Since Einstein's cosmological calculations avoid dealing with the time-variant effects of moving gravitational fields by presuming that all matter in the universe being modelled has zero conventional relative motion, they are not obviously an extension or generalisation of relativity theory to cosmology.

ii ... which, coincidentally, is the same set needed for classical Hawking radiation, relative horizons and Hubble cosmology. [59]

iii ... which would obviously be more efficient than the current system.

7 Conclusions

Modern expanding-universe cosmology is sometimes presented as being an outgrowth and endorsement of Einstein's 1916 general theory. Einstein wrote in 1950, in the fourth appendix to "*Relativity: the Special and the General Theory*", that,

... Hubble showed ... a redshift that increased regularly with the distance of the nebulae. This can be interpreted with regard to our present knowledge only in the sense of Doppler's principle, as an expansive motion of the system of stars in the large – as required, according to Friedman, by the field equations of gravitation. Hubble's discovery can, therefore be considered to some extent as a confirmation of the theory. "

Einstein's claim is not obviously supported by the literature.

- Friedman does not appear to have shown that Einstein's gravitational equations "required" a closed universe, with expansion and expansion redshifts: he also produced other solutions to Einstein's equations in which the universe had positive curvature, negative curvature, or no curvature at all, and in which it was expanding, contracting, or briefly static. Since his solutions covered almost every possibility one could imagine, it was disingenuous and perhaps dishonest to pick out the one option that agreed best with Hubble's experimental data, and count this as a partial experimental confirmation of "the theory".
- If we supplemented Einstein's system with auxiliary arguments that *did* predict a Hubble redshift, then it would be these additional arguments that deserved the credit for the prediction, regardless of whether the underlying gravitational model was right or wrong.
- Further, it is not clear that Einstein's shift-symmetrical system is even *compatible* with a Hubble universe: [ii](#) its natural default predictions are those given in the 1916 paper, of a universe that is infinite, static, and pseudo-Euclidean. [iii](#)
- Lastly, not only is Hubble shift incompatible with the SR Doppler law, and expansion redshift incompatible with the Schwarzschild solution, Hubble expansion contradicts the theme running through Einstein's approach to physics, that physical laws and the equations describing them need to be identical in forward and reversed time. [iv](#) [\[4\]](#)

Hubble redshifts and other expansion-related phenomena suggest a different class of generally-relativistic theory to the SR-centric version presented by Einstein. If we treat these effects as tests of gravitational theory, they must be considered an apparent falsification of Einstein's architecture, design ethos, and equations.

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- ii Modern cosmology requires all three main forms of shift mechanism – motion, gravitation and time-dependent density-change – to disagree with Einstein's 1916 predictions, so that we can only rescue Einstein's system from contradiction by inventing duplicate versions of these effects for cosmology that obey non-Einstein laws, and by studiously avoiding the question of how these two sets of conflicting law are then supposed to be satisfied by a single geometry.
 - iii To achieve a lack of large-scale curvature (total cancellation of gravitational shifts, required by the inherited SR shift relationships), in a universe populated with sources of *positive* curvature, then requires a compensating *negative*-curvature field ... which may be the real motivation for Einstein's 1917 Cosmological Constant.
 - iv If a contemporary time-reversed observer can take a snapshot of our current universe as it appears to *them*, and conclude that the universe is expanding in *their* direction of time, then we have a paradox. If they *cannot* do this, and are forced to agree that the universe should be preferentially expanding in *our* direction of forward time, then the equations that we and they use must be asymmetrical.

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